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DECOMPOSITION OF ACCIDENT LOSS AND DECOUPLED LIABILITY ASSIGNMENT: A CLASS OF NEGLIGENCE RULES

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DECOMPOSITION OF ACCIDENT LOSS AND DECOUPLED LIABILITY ASSIGNMENT: A CLASS OF NEGLIGENCE RULES

PAPIYA GHOSH AND RAJENDRA P. KUNDU

Abstract. This paper is a contribution to the literature on efficient assignment of liabilities for accidental losses arising out of two party interactions involving negative externalities. The objective is to examine the requirements that efficiency imposes on rules for the assignment of liabilities for such losses. We study efficiency properties of a very general class of rules which (i) decompose the loss into two components (specified loss and excess loss);(ii) assigns the entire excess loss to the injurer if she is negligent and to the victim otherwise; and assigns fixed proportions (not necessarily adding up to 1) of the specified loss to the two parties with the possibility of eventually resulting in an assignment of liabilities which can in principle be decoupled. In contrast to existing results we demonstrate that assignment of the specified loss is also important for efficiency and complete decoupling is not inconsistent with efficiency.

Keywords: Liability Rule, negligence, decomposition of loss, decoupled liability, efficiency.
JEL Classification: K13,

1. Introduction

This paper is a contribution to the literature on legal assignment of liabilities for accidental losses arising out of interactions involving negative externalities.[1] The objective is to examine the requirements that efficiency imposes on rules for the assignment of liabilities for such losses. We introduce and analyze the efficiency properties of a new class of rules generalizing the standard negligence rule which is one of the most important rules for the apportionment of accidental losses. Under the negligence rule the entire loss is assigned to the injurer if she is negligent and to the victim otherwise. In contrast, our rules assign only a part of the loss to the injurer if she is negligent and to the victim otherwise and the remaining loss is assigned to the parties in fixed proportions (not necessarily adding up to 1). Our results characterizing efficiency for these rules provide us with new and interesting insights into the requirements that efficiency considerations may impose on the assignment of liabilities for different parts of the accidental loss and also suggest a possible way of mitigating the serious problem that wealth constrained interacting parties may pose for attainment of economic efficiency.

We exploit the framework which is now standard in the significant literature on the efficient assignment of liabilities.[2] In this framework the analysis is usually done in the context of interactions between two risk-neutral parties who are strangers to one another. It is assumed

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[1]The legal remedy of assignment of liabilities between interacting parties is the most important among the alternatives to appropriately designed taxes as a solution to the problem of negative externalities.

[2]Pioneering contributions to this literature came from Calabresi ([3], [4], [5]) and Coase[6] followed by Brown[2], Diamond[7], Posner ([21], [22]), Shavell [25], Jain and Singh[15], Jain([11], [10]), Jain and Kundu[14], Grady ([8], [9]), Kundu and Pal[19] and Kundu and Kaur[18] among others. Brown was the first to present a formal analysis of many of the important rules for assignment of liabilities. The subsequent literature is built upon Brown's formal model which was later on generalized by Jain and Singh[15]. A systematic treatment of the economic analysis of rules for the assignment of liabilities is contained in Jain[12].

that the loss, in case of accident, falls on one of the parties called the victim. The other party is referred to as the injurer. It is also assumed that the probability of accident and the quantum of loss and hence the expected accident loss from this interaction depends on the care levels of the two parties and the social objective is to minimize the total social costs which are defined as the sum of costs of care of the parties involved and the expected accident losses. The assignment of liabilities for the losses in case of occurrence of accident is usually based on the levels of nonnegligence of the victim and the injurer where nonnegligence of the parties are defined with respect to legally specified due care levels which are assumed to be total social cost maximizing. If a party chooses a care level which is less than the due care specified for the party then the party is called negligent, otherwise the party is nonnegligent. The level of nonnegligence of a party is either 1 or the ratio of her actual level of care to the due care level legally specified for her, whichever is lower. A rule for the assignment of liabilities specifies the portions of the loss, in case of occurrence of accident, that the victim and the injurer have to bear for every possible combination of their levels of nonnegligence. An application of a rule is a complete specification of the possible care levels of the two parties, the expected loss function and the due care levels for each of the two parties. The expected cost of a party is defined to be the sum of her cost of care and the expected value of the share of loss to be borne by him under the given rule. It is assumed that the parties know the rule and the application and choose their care levels simultaneously to minimize their expected costs. Thus an application of a given rule is a two-player simultaneous move game of complete information. A rule for assignment of liabilities is efficient if and only if for every game that it can induce there is a Nash equilibrium and every Nash equilibrium is total social costs minimizing. In other words, a rule for assignment of liabilities is said to be efficient if and only if it always induces both parties to choose care levels that minimize total social costs.

In the literature, rules for the assignment of liabilities have been formally conceptualized in different ways and at different levels of generality. The most basic of these formal notions is that of a *liability rule* which assigns the entire loss between the interacting parties on the basis of their levels of nonnegligence. Efficiency analysis of all the standard rules[3] have been done using the notion of a liability rule and it has been established that a liability rule is efficient if and only if it is such that whenever one party is negligent and the other is not the negligent party is made to bear the entire loss.[4] The notion of a liability rule has, subsequently, been generalized in various directions to explore different substantive ideas related to efficient assignment of liabilities. Two of these generalizations have direct bearing on the current paper.

[3]The liability rules which are most widely used by the courts include the rules of *no liability*, *strict liability*, *negligence*, *strict liability with the defence of contributory negligence* and *negligence with the defence of contributory negligence*. These rules have been extensively studied and analyzed in the literature. The assignment of liabilities under these rules are as follows:

(i) *No liability*: the victim always bears the entire loss and injurer bears nothing.
(ii) *Strict liability*: the injurer is always liable for the entire loss and victim bears nothing.
(iii) *Negligence*: if the injurer is negligent then she has to bear the entire loss and victim bears nothing; if the injurer is not negligent then she bears nothing and the entire loss is borne by the victim.
(iv) *Strict liability with the defence of contributory negligence*: if the victim is negligent then she has to bear the entire loss and injurer bears nothing; if the victim is not negligent then she bears nothing and the entire loss is borne by the injurer.
(v) *Negligence with the defence of contributory negligence*: if the injurer is negligent and the victim is not then she has to bear the entire loss and victim bears nothing; otherwise she bears nothing and the entire loss is borne by the victim.

[4]This result is due to Jain and Singh[15].

First, to systematically examine whether or not the assignment of the entire loss is necessary for achieving efficiency, Jain and Kundu[13] introduced a generalization of the standard negligence rule called *(λ, θ)-negligence rule.* A (λ, θ)-negligence rule (i) decomposes the loss into two components: specified loss and excess loss;[5] (ii) apportions the entire specified loss in fixed proportions $(\lambda, 1 - \lambda)$ between the victim and the injurer; and (iii) apportions the entire excess loss to the injurer if she is negligent and to the victim otherwise. Jain and Kundu[13] demonstrate that a (λ, θ)-negligence rule is efficient if and only if the specified loss is such that its expected value is not greater than the expected loss at the levels of care due from the two parties. Thus, it appears that, the assignment of a part of the loss (up to the loss evaluated at due care levels of the parties) does not have any efficiency implications and therefore can by assigned on the basis of other considerations.[6]

The second set of generalizations are related to the idea of coupling in tort law and its implications for efficiency. The assignment of liabilities corresponding to any combination of the levels of nonnegligence of the interacting parties is said to be *coupled* if and only if the interacting parties together are made to bear the full loss and is said to be *decoupled* if and only if the interacting parties together bear less or more than the loss.[7] By definition, the assignment of liabilities under a liability rule is coupled for all combinations of levels of nonnegligence of the parties.[8] Jain [11] introduced the idea of a *hybrid liability rule* under which the assignment of liabilities is either coupled always or decoupled always and demonstrates that every rule under which the liabilities are always decoupled is inefficient. Kundu and Kaur ([16], [17], [16]) study efficiency properties of a class of rules called *liability assignment rules* under which the assignment of liabilities need not be coupled for all possible combinations of the levels of nonnegligence of the interacting parties and demonstrate that the liabilities under an efficient rule has to be coupled when one party is negligent and the other is not.

In this paper we consider a general class of rules called *$((\lambda_1, \lambda_2),\ \theta)$-negligence rules* under which (i) the actual loss is decomposed into two parts: specified loss and excess loss (ii) fixed portions of the specified loss (not necessarily adding up to the the specified loss) are assigned to the two parties and (iii) the entire excess loss is assigned to the injurer if she is negligent and to the victim otherwise. Thus, while the liabilities for the excess loss are always coupled the liabilities for the specified loss are not necessarily so. It has to be noted that, unlike a (λ, θ)-negligence rule, a *$((\lambda_1, \lambda_2),\ \theta)$-negligence rule* can result in an assignment of liabilities which is decoupled. Also note that, unlike in the case of liability assignment rules, under *$((\lambda_1, \lambda_2),\ \theta)$-negligence rules* the assignment of a part of the loss is independent of the level of nonnegligence of the parties. The main result of the paper establishes that a *$((\lambda_1, \lambda_2),\ \theta)$-negligence rule* is efficient if and only if the specified loss is such that its expected value is not greater than the

[5]The specified loss is a multiple of the loss evaluated at the due care levels suitably adjusted to take into account differing probabilities of accidents with different care levels. The amount of the loss over and above specified loss is called the excess loss. Decomposition of the loss and related ideas are more formally defined in section 2.

[6]Jain and Kundu[14] introduced the notion of a (λ, θ)-decomposed liability rule. A (λ, θ)-decomposed liability rule (i) decomposes the loss into two components (specified loss and excess loss);(ii) apportions the entire excess loss between the victim and the injurer on the basis of their levels of nonnegligence; and (iii) apportions the entire specified loss in fixed proportions between the victim and the injurer. Jain and Kundu[14] demonstrate that as long as the specified loss is such that its expected value is not greater than the expected optimal loss a (λ, θ)-decomposed liability rule is efficient if and only if it is such that whenever one party is negligent and the other is not the negligent party is made to bear the entire excess loss.

[7]Liabilities may be decoupled when (i) a part of the amount imposed on the injurer is allocated to some state or court administered fund or (ii) the victim is compensated out of some special fund set up for this purpose. In the US several states have split recovery statutes which necessitate compulsory state sharing of damages. Victims are often compensated by the state if the injurer is insolvent and judgement-proof. See Polinsky and Che[20] and Sharkey[23].

[8]The same holds true for a (λ, θ)-decomposed liability rule.

loss at the levels of care due from the two parties and the shares of the specified loss to be borne by the two parties add upto less than or equal to 1. Thus, it turns out, that both the size of the two parts of the loss and the assignment of liabilities for the specified loss have implications for efficiency.

The paper is organized as follows: The model is presented in Section 2. All definitions and assumptions are stated here and are illustrated with appropriate examples. Section 3 contains the main results of the paper in the form of Theorems 1-3 and also contains the intermediate results (Propositions 1-5) which are used to prove theorems 1 and 2. Section 4 discusses some implications of the results and Section 5 concludes the paper.

2. Model

We consider interactions between two parties (generically called party i where $i \in \{1,2\}$), assumed to be strangers to each other, which can result in an accidental harm falling on party 1. We'll refer to party 1 as the victim and party 2 as the injurer. It is assumed that the probability of accident and the magnitude of harm in case of an accident depends on the level of care that the parties might choose to take. Let $a_i \geq 0$ be the index of the level of care taken by party i and let $A_i = \{a_i \mid a_i \geq 0$ be the index of some feasible level of care which can be taken by party $i\}$ be the set of all possible levels of care by party i. We assume that

$0 \in A_i$. (A1)

We denote by $c_i(a_i)$ the cost to party i of care level a_i. Let $C_i = \{c_i(a_i) \mid a_i \in A_i\}$. We assume

$c_i(0) = 0$. (A2)

We also assume that

c_i is a strictly increasing function of a_i. (A3)

In view of (A2) and (A3) it follows that $(\forall c_i \in C_i)(c_i \geq 0)$.

A consequence of (A3) is that c_i itself can be taken to be an index of the level of care taken by party i.

Let $\pi : C_1 \times C_2 \mapsto [0,1]$ denote the probability of occurrence of accident and $H : C_1 \times C_2 \mapsto \Re_+$ denote the loss in case of occurrence of accident. Let $L : C_1 \times C_2 \mapsto \Re_+$ be defined as: $L(c_1, c_2) = \pi(c_1, c_2)H(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. L is thus the expected loss due to accident.

We assume:

π and H are non-increasing in c_1 and c_2. (A4)

(A4) implies that L is non-increasing in c_1 and c_2.

We define the total social cost of the interaction between the two parties, $T : C_1 \times C_2 \mapsto \Re_+$, as $T(c_1, c_2) = c_1 + c_2 + L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. Let $M = \{(c_1', c_2') \in C_1 \times C_2 \mid (\forall (c_1, c_2) \in C_1 \times C_2)[T(c_1', c_2') \leq T(c_1, c_2)\}$. Thus M is the set of all costs of care profiles which are total social cost minimizing. It will be assumed that:

C_1, C_2, π and H are such that M is nonempty. (A5)

A specification of C_1, C_2 and L satisfying (A1)-(A5) along with a choice of $(c_1^*, c_2^*) \in M$ completely describes a context for assignment of liabilities. Any such context will be denoted by ω and the set of all ω will be denoted by Ω. We shall denote $H(c_1^*, c_2^*), \pi(c_1^*, c_2^*)$, $L(c_1^*, c_2^*)$ and $T(c_1^*, c_2^*)$ by H^*, π^*, L^* and T^* respectively. Similarly, we shall denote $H(\bar{c}_1, \bar{c}_2)$, $L(\bar{c}_1, \bar{c}_2)$ and $T(\bar{c}_1, \bar{c}_2)$ by $\overline{H}, \overline{L}$ and $\overline{T}$ respectively. H^* will be termed as optimal loss, $H^* \frac{\pi^*}{\pi(c_1, c_2)}$ as adjusted optimal loss and L^* as optimal expected loss.

2.1. Decomposition of loss.

Let $\theta \geq 0$. Given ω, we define functions $G : C_1 \times C_2 \mapsto \Re_+$ and $S : C_1 \times C_2 \mapsto \Re_+$ as follows:

$$
\begin{aligned}
G(c_1,c_2) &= \theta H^* \tfrac{\pi^*}{\pi(c_1,c_2)} \quad \text{if} \quad \pi(c_1,c_2) \neq 0 \text{ and } H(c_1,c_2) > \theta H^* \tfrac{\pi^*}{\pi(c_1,c_2)} \\
&= H(c_1,c_2) \quad \text{otherwise.} \\
S(c_1,c_2) &= H(c_1,c_2) - \theta H^* \tfrac{\pi^*}{\pi(c_1,c_2)} \quad \text{if } \pi(c_1,c_2) \neq 0 \text{ and } H(c_1,c_2) > \theta H^* \tfrac{\pi^*}{\pi(c_1,c_2)} \\
&= 0 \quad \text{otherwise.}
\end{aligned}
$$

If total accident loss is greater than θ times the adjusted optimal loss then the accident loss is decomposed into two parts: the excess of accident loss over θ times the adjusted optimal loss being the S part, to be called excess loss, and the θ times the adjusted optimal loss being the G part, to be called the specified loss; otherwise the entire accident loss is equated with G.
Hence, $S(c_1,c_2) = \max\{H(c_1,c_2) - \theta H^* \frac{\pi^*}{\pi(c_1,c_2)}, 0\}$ and $G(c_1,c_2) = \min\{\theta H^* \frac{\pi^*}{\pi(c_1,c_2)}, H(c_1,c_2)\}$. It should be noted that S and G always sum to total accident loss. Thus, (S,G) constitutes a decomposition of accident loss H. If adjusted optimal loss is positive then by varying θ between 0 and a sufficiently large positive number all possible decompositions of accident loss in two parts can be obtained. If adjusted optimal loss is 0 then, regardless of the value of θ, S would be equal to the total accident loss and G equal to 0.

Let $R : C_1 \times C_2 \mapsto \Re_+$ and $F : C_1 \times C_2 \mapsto \Re_+$ be defined as follows:

$R(c_1,c_2) = \pi(c_1,c_2)S(c_1,c_2)$ and

$F(c_1,c_2) = \pi(c_1,c_2)G(c_1,c_2)$.

Therefore $R(c_1,c_2) = \max\{L(c_1,c_2) - \theta L^*, 0\}$ and $F(c_1,c_2) = \min\{\theta L^*, L(c_1,c_2)\}$. Thus, (R,F) constitutes a decomposition of expected accident loss L. Consistent with the nomenclature of G and S, $F(c_1,c_2)$ will be referred to as the expected specified loss and $R(c_1,c_2)$ as the expected excess loss.
We shall denote $R(c_1^*,c_2^*), F(c_1^*,c_2^*), R(\bar{c}_1,\bar{c}_2)$ and $F(\bar{c}_1,\bar{c}_2)$ by $R^*, F^*, \overline{R}$ and $\overline{F}$ respectively.

Remark 1. *If $\theta = 0$ then $R(c_1,c_2) = L(c_1,c_2)$ and $F(c_1,c_2) = 0$ for all $(c_1,c_2) \in C_1 \times C_2$.*

Remark 2. *If $0 < \theta \leq 1$ then $R^* = (1-\theta)L^*$ and $F^* = \theta L^*$. If $\theta > 1$ then $R^* = 0$ and $F^* = L^*$.*

Remark 3. *Suppose $0 < \theta \leq 1$. If $L(c_1,c_2) > L^*$ then $R(c_1,c_2) = L(c_1,c_2) - \theta L^*$ and $F(c_1,c_2) = \theta L^*$. If $L(c_1,c_2) \leq L^*$ and $L(c_1,c_2) > \theta L^*$ then $R(c_1,c_2) = L(c_1,c_2) - \theta L^*, F(c_1,c_2) = \theta L^*$. If $L(c_1,c_2) \leq L^*$ and $L(c_1,c_2) \leq \theta L^*$ then $R(c_1,c_2) = 0, F(c_1,c_2) = L(c_1,c_2)$.*

2.2. **Negligence.** Given ω, we define functions $p_i : C_i \mapsto [0,1]$as follows:

$$
\begin{aligned}
p_i(c_i) &= \tfrac{c_i}{c_i^*} \quad \text{if } c_i < c_i^* \\
&= 1 \quad \text{otherwise.}
\end{aligned}
$$

$p_i(c_i)$ would be interpreted as the proportion of nonnegligence of party i. If $p_i(c_i) = 1$, party i would be called nonnegligent; and if $p_i(c_i) < 1$, party i would be called negligent.
If there is a legally specified due care level for party i then c_i^* in the definition of p_i would be taken to be identical with the legally specified due care level.[9] If no due care level is legally specified for party i then c_i^* used in the definition of p_i can be taken to be any $c_i^* \in C_i$ subject

[9]Note that specification of a due care level for a party may or may be required depending on the rule for the assignment of liabilities. For example, the strict liability rule and the no liability rule do require a due care level for either of the parties, negligence rule and the rule of strict liability with the defence of contributory negligence require a due care level for only one of the two parties and rule of negligence with the defence of contributory negligence requires a due care level for each party.

to the requirement that $(c_1^*, c_2^*) \in M$. Thus in all cases, for each party i, c_i^* would denote the legally binding due care level for party i whenever the idea of legally binding due care level for party i is applicable.[10] If there is a legally specified due care level for party i then $p_i(c_i) = 1$ would mean that party i is taking at least the due care and $p_i(c_i) < 1$ would mean that party i is taking less than due care.

2.3. **A class of negligence rules.** A $((\lambda_1, \lambda_2), \theta)$-negligence rule $\mathbf{g}$ is a rule which assigns the specified loss to the victim and the injurer in fixed proportions λ_1 and λ_2 respectively, and assigns the entire excess loss to the injurer if she is negligent and to the victim otherwise. More formally, $((\lambda_1, \lambda_2), \theta)$-negligence rule is a function $\mathbf{g} : [0,1]^2 \mapsto [0,1]^2 \times [0,1]^2$ such that $\mathbf{g}(p_1, p_2) = [(\lambda_1, \lambda_2), (x_1, x_2)]$ where

$$\begin{array}{rcll} (x_1(p_1,p_2), x_2(p_1,p_2)) & = & (0,\,1) & \text{if } p_2 < 1 \\ & = & (1,\,0) & \text{otherwise.} \end{array}$$

Here λ_i represents the fixed portion of the specified loss to be borne by party i and $x_i = x_i(p_1, p_2)$ represents the portion of the excess loss to be borne by party i.[11]

It should be noted that a $((\lambda_1, \lambda_2), \theta)$-negligence rule is a generalization of the standard negligence rule. If $\theta = 0$ then the assignment of liabilities under a $((\lambda_1, \lambda_2), \theta)$-negligence rule is identical to the assignment of liabilities under the standard negligence rule.

2.4. **Efficiency.** Let $\mathbf{g}$ be any $((\lambda_1, \lambda_2), \theta)$-negligence rule and $\omega \in \Omega$ be any application of $\mathbf{g}$. If the victim chooses c_1, the injurer chooses c_2 and the accident occurs then the actual loss will be $H(c_1, c_2)$. According to $\mathbf{g}$ the victim will be made to bear $\lambda_1 G(c_1, c_2) + x_1(p_1(c_1), p_2(c_2))S(c_1, c_2)$ and the injurer will be liable for $\lambda_2 G(c_1, c_2) + x_2(p_1(c_1), p_2(c_2))S(c_1, c_2)$. Thus, $E_1 : C_1 \times C_2 \mapsto \Re_+$ defined as: $E_1(c_1, c_2) = c_1 + \lambda_1 F(c_1, c_2) + x_1(p_1(c_1), p_2(c_2))R(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$ would give us the expected costs to the victim and $E_2 : C_1 \times C_2 \mapsto \Re_+$ defined as: $E_2(c_1, c_2) = c_2 + \lambda_2 F(c_1, c_2) + x_2(p_1(c_1), p_2(c_2))R(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$ would give us the expected costs to the injurer. We assume that for all $(c_1, c_2), (c_1', c_2') \in C_1 \times C_2$, the victim considers (c_1, c_2) to be at least as good as (c_1', c_2') iff $E_1(c_1, c_2) \leq E_1(c_1', c_2')$ and the injurer considers (c_1, c_2) to be at least as good as (c_1', c_2') iff $E_2(c_1, c_2) \leq E_2(c_1', c_2')$. We also assume that the parties have complete information about the rule and the application and choose their respective care levels simultaneously to minimize their own expected costs. Thus an application of a $((\lambda_1, \lambda_2), \theta)$-negligence rule is a two-player simultaneous move game of complete information in which the strategies are the feasible levels of care and the payoffs are the expected costs.[12] We shall denote the game induced by $((\lambda_1, \lambda_2), \theta)$-negligence rule, $\mathbf{g}$ in application ω by $(\mathbf{g}, \omega)$ and, whenever possible, represent it by the corresponding payoff matrix.

A $((\lambda_1, \lambda_2), \theta)$-negligence rule, $\mathbf{g}$ is said to be efficient for ω iff (i) $(\exists (c_1, c_2) \in C_1 \times C_2)[(c_1, c_2)$ is a Nash equilibrium of $(\mathbf{g}, \omega)]$ and (ii) $(\forall (c_1, c_2) \in C_1 \times C_2)[$If (c_1, c_2) is a Nash equilibrium of $(\mathbf{g}, \omega)$ then $(c_1, c_2) \in M]$.[13] A $((\lambda_1, \lambda_2), \theta)$-negligence rule, $\mathbf{g}$ is said to be efficient for Ω iff it is efficient for every $\omega \in \Omega$. In other words, a $((\lambda_1, \lambda_2), \theta)$-negligence rule, $\mathbf{g}$ is said to be efficient for Ω iff for every application $\omega \in \Omega$ (i) there exists a Nash equilibrium of the game $(\mathbf{g}, \omega)$ and (ii) every Nash equilibrium of $(\mathbf{g}, \omega)$ minimizes T.

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[10] Thus, implicitly it is being assumed that the legally specified due care levels are in all cases consistent with the objective of total social cost minimization.

[11] Note that a $((\lambda_1, \lambda_2), \theta)$-negligence rule can be defined as a function of the level of nonnegligence of the injurer. In order to make the comparison of our results with some of the existing ones simple we define a $((\lambda_1, \lambda_2), \theta)$-negligence rule as a function of the nonnegligence of the two parties.

[12] It has to be noted that the payoffs are non-positive.

[13] Only pure strategy Nash equilibria are considered in this paper.

2.5. **Examples.** The following examples illustrate the concepts introduced so far. While Example 1 gives an application satisfying assumptions (A1)-(A5) and shows the decomposition of the loss function for two different values of θ, Examples 2-4 examine the efficiency of three different $((\lambda_1, \lambda_2), \theta)$- negligence rules for the application in Example 1.

Example 1. *Let $C_1 = C_2 = \{0, 2, 4\}$ and let the expected loss be as given in Table 1.*

		c_2: 0	2	4
c_1	0	20	16	15
	2	16	12	11
	4	15	11	9

TABLE 1. Application ω_1

It can be easily checked that $M = \{(2, 2)\}$. Let $(c_1^*, c_2^*) = (2, 2)$. Thus C_1, C_2 given above and $L(c_1, c_2)$ specified as in table 1 constitutes an application in Ω.[14] *We denote this application as ω_1. $(c_1^*, c_2^*) = (2, 2)$ implies that $p_1(0) = 0, p_1(2) = p_1(4) = 1$ and $p_2(0) = 0, p_2(2) = p_2(4) = 1$ For $\theta = 1$ the decomposition of the loss function in ω_1 is given in Table 2.*

		c_2: 0	2	4
c_1	0	$(12, 8)$	$(12, 4)$	$(12, 3)$
	2	$(12, 4)$	$(12, 0)$	$(11, 0)$
	4	$(12, 3)$	$(11, 0)$	$(9, 0)$

TABLE 2. ω_1: (F,R) matrix with $\theta = 1$

For $\theta = 2$ the decomposition of the loss function in ω_1 is given in Table 3.

		c_2: 0	2	4
c_1	0	$(20, 0)$	$(16, 0)$	$(15, 0)$
	2	$(16, 0)$	$(12, 0)$	$(11, 0)$
	4	$(15, 0)$	$(11, 0)$	$(9, 0)$

TABLE 3. ω_1: (F,R) matrix with $\theta = 2$

Note that the first entry in each cell of Tables 2 and 3 denote the expected specified loss and the second entry denote the expected excess loss.

Example 2. *Let $\mathbf{g}_1$ be the ((1/2, 1/2), 1)-negligence rule. Consider ω_1 as an application of $\mathbf{g}_1$. The payoff matrix of the game induced by $\mathbf{g}_1$ is given in Table 4.*
$(2, 2)$, the unique total social cost minimizing configuration of care levels, is also the unique Nash equilibrium of $(\mathbf{g}_1, \omega_1)$. Therefore $\mathbf{g}_1$ is efficient for ω_1.[15]

[14]We shall, simply, refer to a table representing expected loss function as an application.
[15]Theorem 2 establishes that $\mathbf{g}_1$ is efficient for every application in Ω.

c_1 \ c_2	0	2	4
0	$(\underline{6}, 14)$	$(10, \underline{8})$	$(9, 10)$
2	$(8, 10)$	$(\underline{8}, \underline{8})$	$(\underline{7.5}, 9.5)$
4	$(10, 9)$	$(9.5, \underline{7.5})$	$(8.5, 8.5)$

TABLE 4. $(\mathbf{g}_1, \omega_1)$

Example 3. *Let* $\mathbf{g}_2$ *be the ((1, 1), 1)-negligence rule. Consider* ω_1 *as an application of* $\mathbf{g}_2$. *The payoff matrix of the game induced by* $\mathbf{g}_2$ *is given in Table 5.*

c_1 \ c_2	0	2	4
0	$(\underline{12}, 20)$	$(16, \underline{14})$	$(15, 16)$
2	$(14, 16)$	$(\underline{14}, \underline{14})$	$(\underline{13}, 15)$
4	$(16, 15)$	$(15, \underline{13})$	$(\underline{13}, \underline{13})$

TABLE 5. $(\mathbf{g}_2, \omega_1)$

$(4, 4) \notin M$ *is a Nash equilibrium of* $(\mathbf{g}_2, \omega_1)$. *Therefore* $\mathbf{g}_2$ *is not efficient for* ω_1.

Example 4. *Let* $\mathbf{g}_3$ *be the ((1/2, 1/2), 2)-negligence rule. Consider* ω_1 *as an application of* $\mathbf{g}_3$. *The payoff matrix of the game induced by* $\mathbf{g}_3$ *is given in Table 6.*

c_1 \ c_2	0	2	4
0	$(\underline{10}, \underline{10})$	$(\underline{8}, \underline{10})$	$(\underline{7.5}, 11.5)$
2	$(\underline{10}, \underline{8})$	$(\underline{8}, \underline{8})$	$(\underline{7.5}, 9.5)$
4	$(11.5, \underline{7.5})$	$(9.5, \underline{7.5})$	$(8.5, 8.5)$

TABLE 6. $(\mathbf{g}_3, \omega_1)$

$(0, 0) \notin M$ *is a Nash equilibrium of* $(\mathbf{g}_3, \omega_1)$. *Therefore* $\mathbf{g}_3$ *is not efficient for* ω_1.

2.6. **Decoupled assignment of liabilities.** The assignment of liabilities corresponding to any combination of the levels of nonnegligence of the interacting parties is said to be coupled if and only if the liability imposed on the injurer is equal to the payment to the victim and to be decoupled if the two amounts are unequal. Thus, if the assignment of liabilities is coupled then the interacting parties parties together are made to bear the full loss and if the assignment of liabilities is decoupled then the interacting parties together bear less or more than the total loss. Note that, under a $((\lambda_1, \lambda_2), \theta)$-negligence rule the injurer and the victim always bear the full excess loss together but they may have to bear more or less than the full specified loss depending on the values of λ_1, λ_2 and θ. Thus, under a $((\lambda_1, \lambda_2), \theta)$-negligence rule, the assignment of liabilities for the entire loss is either always coupled or always decoupled.

Remark 4. *The assignment of liabilities under a* $((\lambda_1, \lambda_2), 0)$*-negligence rule is always coupled.*

Remark 5. *Suppose* $0 < \theta$. *If* $\lambda_1 + \lambda_2 = 1$ *then the assignment of liabilities under a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule is always coupled. If* $\lambda_1 + \lambda_2 \neq 1$ *then the assignment of liabilities under a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule is always decoupled. In particular, if* $\lambda_1 + \lambda_2 < 1$ *then the parties together bear less than the total accident loss and if* $\lambda_1 + \lambda_2 > 1$ *then they together bear more than the total accident loss.*

Example 5. *The assignment of liabilities under* $\mathbf{g}_1$ *and* $\mathbf{g}_3$ *are always coupled. Under* $\mathbf{g}_2$ *the assignment of liabilities is always decoupled.*

3. Results

In this section we present our analysis of the entire class of $((\lambda_1, \lambda_2), \theta)$-negligence rules. Depending on the value of the decomposition parameter θ, we divide the entire class of rules into 3 subclasses and study efficiency properties of rules in each of these subclasses. We find that while every $((\lambda_1, \lambda_2), \theta)$-negligence rule is efficient for $\theta = 0$, none is efficient for $\theta > 1$; and for $0 < \theta \leq 1$ the efficiency or otherwise of a $((\lambda_1, \lambda_2), \theta)$-negligence rule depends on the value of $\lambda_1 + \lambda_2$. Our main results are presented as Theorems 1-3. Theorem 1 states that every $((\lambda_1, \lambda_2), 0)$-negligence rule is efficient. Theorem 2 states that for $0 < \theta \leq 1$ a $((\lambda_1, \lambda_2), \theta)$-negligence rule is efficient iff $\lambda_1 + \lambda_2 \leq 1$. Theorem 3 states that no $((\lambda_1, \lambda_2), \theta)$-negligence rule is efficient for $\theta > 1$.

In what follows we state and prove five intermediate results to prove Theorems 1 and 2 (Propositions 1 and 2 together establish Theorem 1 and Propositions 3-5 together establish Theorem 2) and provide a proof of Theorem 3.

Proposition 1. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule and consider any application* $\omega \in \Omega$ *of* $\mathbf{g}$*. If* $\theta = 0$ *then* (c_1^*, c_2^*) *is a Nash equilibrium of* $(\mathbf{g}, \omega)$*.*

Proof. Let $\mathbf{g}$ be a $((\lambda_1, \lambda_2), \theta)$-negligence rule where $\theta = 0$. Consider any application $\omega \in \Omega$ of $\mathbf{g}$. Suppose (c_1^*, c_2^*) is not a Nash equilibrium of $(\mathbf{g}, \omega)$. This implies that, there exists $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ or there exists $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.

Note that $\theta = 0$ implies $F(c_1, c_2) = 0$ and $R(c_1, c_2) = L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. We know that $E_1(c_1^*, c_2^*) = c_1^* + L^*$ and $E_2(c_1^*, c_2^*) = c_2^*$. Also, $E_1(c_1', c_2^*) = c_1' + L(c_1', c_2^*)$. $c_2' < c_2^* \to E_2(c_1^*, c_2') = c_2' + L(c_1^*, c_2')$ and $c_2' \geq c_2^* \to E_2(c_1^*, c_2') = c_2'$.

$E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $c_1' + L(c_1', c_2^*) < c_1^* + L^*$ which implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$, a contradiction to the fact that $(c_1^*, c_2^*) \in M$. Therefore, there does not exist $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$.

If $c_2' < c_2^*$ then $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ implies $c_2' + L(c_1^*, c_2') < c_2^*$ which implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$, a contradiction to the fact that $(c_1^*, c_2^*) \in M$. If $c_2' \geq c_2^*$ then $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ implies $c_2' < c_2^*$, a contradiction. Thus, in all cases, $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ leads to a contradiction. Therefore, there does not exist $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.

Hence, (c_1^*, c_2^*) is a Nash equilibrium of (g, ω). □

Proposition 2. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule and consider any application* $\omega \in \Omega$ *of* $\mathbf{g}$*. If* $\theta = 0$ *then ,* $(\overline{c}_1, \overline{c}_2)$ *is a Nash equilibrium of* $(\mathbf{g}, \omega)$ *implies* $(\overline{c}_1, \overline{c}_2) \in M$*.*

Proof. Let $\mathbf{g}$ be a $((\lambda_1, \lambda_2), \theta)$-negligence rule where $\theta = 0$. Consider any application $\omega \in \Omega$ of $\mathbf{g}$.

If $(\overline{c}_1, \overline{c}_2)$ is a Nash equilibrium then $\overline{E}_1 \leq E_1(c_1^*, \overline{c}_2)$ and $\overline{E}_2 \leq E_2(\overline{c}_1, c_2^*)$ and, therefore,

$$\overline{E}_1 + \overline{E}_2 \leq E_1(c_1^*, \overline{c}_2) + E_2(\overline{c}_1, c_2^*). \tag{2.1}$$

Note that $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + (\overline{x}_1 + \overline{x}_2)\overline{R} + (\lambda_1 + \lambda_2)\overline{F} = \overline{c}_1 + \overline{c}_2 + \overline{L} = \overline{T}$ and $E_1(c_1^*, \overline{c}_2) + E_2(\overline{c}_1, c_2^*)$ $= c_1^* + x_1(1, \overline{p}_2)R(c_1^*, \overline{c}_2) + \lambda_1 F(c_1^*, \overline{c}_2) + c_2^* + \lambda_2 F(\overline{c}_1, c_2^*) = c_1^* + c_2^* + x_1(1, \overline{p}_2)L(c_1^*, \overline{c}_2)$.

Also note that, if $\overline{c}_2 < c_2^*$ then $x_1(1, \overline{p}_2) = 0$ and if $\overline{c}_2 \geq c_2^*$ then $x_1(1, \overline{p}_2) = 1$.

If $\overline{c}_2 < c_2^*$ then (2.1) implies $\overline{T} \leq c_1^* + c_2^*$ which implies $\overline{T} \leq T^*$.
If $\overline{c}_2 \geq c_2^*$ then (2.1) implies $\overline{T} \leq c_1^* + c_2^* + L(c_1^*, \overline{c}_2)$ which, in view of the fact that $\overline{c}_2 \geq c_2^* \rightarrow L(c_1^*, \overline{c}_2) \leq L^*$ implies $\overline{T} \leq T^*$.

Thus, in all cases (2.1) implies $\overline{T} \leq T^*$. This, in view of the fact that $(c_1^*, c_2^*) \in M$, implies $(\overline{c}_1, \overline{c}_2) \in M$.

□

Theorem 1. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule. If* $\theta = 0$ *then* $\mathbf{g}$ *is efficient.*

Proof. Immediate from Propositions 1 and 2. □

Proposition 3. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule and consider any application* $\omega \in \Omega$ *of* $\mathbf{g}$*. If* $0 < \theta \leq 1$ *and* $\lambda_1 + \lambda_2 \leq 1$ *then* (c_1^*, c_2^*) *is a Nash equilibrium of* $(\mathbf{g}, \omega)$*.*

Proof. Let $\mathbf{g}$ be a $((\lambda_1, \lambda_2), \theta)$-negligence rule, where $0 < \theta \leq 1$ and $\lambda_1 + \lambda_2 \leq 1$. Consider any application $\omega \in \Omega$ of $\mathbf{g}$. Suppose (c_1^*, c_2^*) is not a Nash equilibrium of $(\mathbf{g}, \omega)$. This implies that, there exists $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ or there exists $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.

Note that $\theta \leq 1$ implies $E_1(c_1^*, c_2^*) = c_1^* + L^* - \theta L^* + \lambda_1 \theta L^*$ and $E_2(c_1^*, c_2^*) = c_2^* + \lambda_2 \theta L^*$. $c_1' < c_1^* \rightarrow E_1(c_1', c_2^*) = c_1' + L(c_1', c_2^*) - \theta L^* + \lambda_1 \theta L^*$; $c_1' \geq c_1^*$ and $L(c_1', c_2^*) > \theta L^* \rightarrow E_1(c_1', c_2^*) = c_1' + L(c_1', c_2^*) - \theta L^* + \lambda_1 \theta L^*$; $c_1' \geq c_1^*$ and $L(c_1', c_2^*) \leq \theta L^* \rightarrow E_1(c_1', c_2^*) = c_1' + \lambda_1 L(c_1', c_2^*)$. $c_2' < c_2^* \rightarrow E_2(c_1^*, c_2') = c_2' + L(c_1^*, c_2') - \theta L^* + \lambda_2 \theta L^*$; $c_2' \geq c_2^* \wedge L(c_1^*, c_2') > \theta L^* \rightarrow E_2(c_1^*, c_2') = c_2' + \lambda_2 \theta L^*$; $c_2' \geq c_2^* \wedge L(c_1^*, c_2') \leq \theta L^* \rightarrow E_2(c_1^*, c_2') = c_2' + \lambda_2 L(c_1^*, c_2')$.

Suppose, there exists $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$.
If $c_1' < c_1^*$ or $[c_1' \geq c_1^*$ and $L(c_1', c_2^*) > \theta L^*]$ then $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $c_1' + L(c_1', c_2^*) - \theta L^* + \lambda_1 \theta L^* < c_1^* + L^* - \theta L^* + \lambda_1 \theta L^*$ which implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$.
If $c_1' \geq c_1^*$ and $L(c_1', c_2^*) \leq \theta L^*$ then $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $c_1' + \lambda_1 L(c_1', c_2^*) < c_1^* + L^* - \theta L^* + \lambda_1 \theta L^*$ which implies $c_1' + L(c_1', c_2^*) < c_1^* + L^* - \theta L^* + \theta L^*$ which implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$.
Thus, in all cases, $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$ implies $T(c_1', c_2^*) < T(c_1^*, c_2^*)$ which contradicts the fact that $(c_1^*, c_2^*) \in M$. Therefore, there does not exist $c_1' \in C_1$ such that $E_1(c_1', c_2^*) < E_1(c_1^*, c_2^*)$.

Suppose, there exists $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.
If $c_2' < c_2^*$ then $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ implies $c_2' + L(c_1^*, c_2') - \theta L^* + \lambda_2 \theta L^* < c_2^* + \lambda_2 \theta L^*$ which implies $T(c_1^*, c_2') < T(c_1^*, c_2^*)$ which contradicts the fact that $(c_1^*, c_2^*) \in M$.
If $c_2' \geq c_2^*$ and $L(c_1^*, c_2') > \theta L^*$ then $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ implies $c_2' + \lambda_2 \theta L^* < c_2^* + \lambda_2 \theta L^*$ which is a contradiction.
If $c_2' \geq c_2^*$ and $L(c_1^*, c_2') \leq \theta L^*$ then $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ implies $c_2' + \lambda_2 L(c_1^*, c_2') < c_2^* + \lambda_2 \theta L^*$ which, in view of the fact that $\lambda_2 \leq 1$, implies which implies $T(c_1^*, c_2') < T(c_1^*, c_2^*)$.

Thus, in all cases, $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$ leads to a contradiction. Therefore, there does not exist $c_2' \in C_2$ such that $E_2(c_1^*, c_2') < E_2(c_1^*, c_2^*)$.

Hence, (c_1^*, c_2^*) is a Nash equilibrium of (g, ω).

□

Proposition 4. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule and consider any application* $\omega \in \Omega$ *of* $\mathbf{g}$*. If* $0 < \theta \leq 1$ *and* $\lambda_1 + \lambda_2 \leq 1$ *then,* $(\overline{c}_1, \overline{c}_2)$ *is a Nash equilibrium of* $(\mathbf{g}, \omega)$ *implies* $(\overline{c}_1, \overline{c}_2) \in M$*.*

Proof. Let $\mathbf{g}$ be a $((\lambda_1, \lambda_2), \theta)$-negligence rule, where $0 < \theta \leq 1$ and $\lambda_1 + \lambda_2 \leq 1$. Consider any application $\omega \in \Omega$ of $\mathbf{g}$.

If $(\overline{c}_1, \overline{c}_2)$ is a Nash equilibrium then $\overline{E}_1 \leq E_1(c_1^*, \overline{c}_2)$ and $\overline{E}_2 \leq E_2(\overline{c}_1, c_2^*)$ and, therefore,
$\overline{E}_1 + \overline{E}_2 \leq E_1(c_1^*, \overline{c}_2) + E_2(\overline{c}_1, c_2^*)$. (4.1)

Note that $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + (\overline{x}_1 + \overline{x}_2)\overline{R} + (\lambda_1 + \lambda_2)\overline{F} = \overline{c}_1 + \overline{c}_2 + \overline{R} + (\lambda_1 + \lambda_2)\overline{F}$ and $E_1(c_1^*, \overline{c}_2) + E_2(\overline{c}_1, c_2^*) = c_1^* + x_1(1, \overline{p}_2)R(c_1^*, \overline{c}_2) + \lambda_1 F(c_1^*, \overline{c}_2) + c_2^* + \lambda_2 F(\overline{c}_1, c_2^*)$.

The values of $x_1(1, \overline{p}_2), R(c_1^*, \overline{c}_2), F(c_1^*, \overline{c}_2)$ depends on the relation between $\overline{c}_2$ and c_2^*; value of $F(\overline{c}_1, c_2^*)$ depends on the relation between $\overline{c}_1$ and c_1^*; values of $\overline{x}_1, \overline{x}_2, \overline{R}, \overline{F}$ depend on the relation between $\overline{c}_1$ and c_1^* and the relation between $\overline{c}_2$ and c_2^*.

$\overline{c}_1 < c_1^*$ implies $L(\overline{c}_1, c_2^*) \geq L^* \geq \theta L^*, F(\overline{c}_1, c_2^*) = \theta L^*$ and therefore $E_2(\overline{c}_1, c_2^*) = c_2^* + \lambda_2 \theta L^*$.(4.2)

$\overline{c}_1 \geq c_1^*$ and $L(\overline{c}_1, c_2^*) > \theta L^*$ implies $R(\overline{c}_1, c_2^*) = L(\overline{c}_1, c_2^*) - \theta L^*, F(\overline{c}_1, c_2^*) = \theta L^*$ and therefore $E_2(\overline{c}_1, c_2^*) = c_2^* + \lambda_2 \theta L^*$. (4.3)

$\overline{c}_1 \geq c_1^*$ and $L(\overline{c}_1, c_2^*) \leq \theta L^*$ implies $F(\overline{c}_1, c_2^*) = L(\overline{c}_1, c_2^*)$ and therefore, $E_2(\overline{c}_1, c_2^*) = c_2^* + \lambda_2 L(\overline{c}_1, c_2^*)$. (4.4)

$\overline{c}_2 < c_2^*$ implies $x_1(1, \overline{p}_2) = 0, L(c_1^*, \overline{c}_2) \geq L^* \geq \theta L^*, F(c_1^*, \overline{c}_2) = \theta L^*$ and therefore, $E_1(c_1^*, \overline{c}_2) = c_1^* + \lambda_1 \theta L^*$. (4.5)

$\overline{c}_2 \geq c_2^*$ and $L(c_1^*, \overline{c}_2) > \theta L^*$ implies $x_1(p_1^*, \overline{p}_2) = 1, R(c_1^*, \overline{c}_2) = L(c_1^*, \overline{c}_2) - \theta L^*, F(c_1^*, \overline{c}_2) = \theta L^*$ and therefore, $E_1(c_1^*, \overline{c}_2) = c_1^* + L(c_1^*, \overline{c}_2) - \theta L^* + \lambda_1 \theta L^*$. (4.6)

$\overline{c}_2 \geq c_2^*$ and $L(c_1^*, \overline{c}_2) \leq \theta L^*$ implies $R(c_1^*, \overline{c}_2) = 0, F(c_1^*, \overline{c}_2) = L(c_1^*, \overline{c}_2)$ and therefore, $E_1(c_1^*, \overline{c}_2) = c_1^* + \lambda_1 L(c_1^*, \overline{c}_2)$. (4.7)

$\overline{c}_1 < c_1^*$ and $\overline{c}_2 < c_2^*$ implies $\overline{R} = \overline{L} - \theta L^*, \overline{F} = \theta L^*$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + \overline{L} - \theta L^* + (\lambda_1 + \lambda_2)\theta L^*$. (4.8)

$\overline{c}_1 < c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} > \theta L^*$ implies $\overline{R} = \overline{L} - \theta L^*, \overline{F} = \theta L^*$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + \overline{L} - \theta L^* + (\lambda_1 + \lambda_2)\theta L^*$. (4.9)

$\overline{c}_1 < c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} \leq \theta L^*$ implies $\overline{R} = 0, \overline{F} = \overline{L}$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L}$. (4.10)

$\overline{c}_1 \geq c_1^*, \overline{c}_2 < c_2^*$ and $\overline{L} > \theta L^*$ implies $\overline{R} = \overline{L} - \theta L^*, \overline{F} = \theta L^*$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + \overline{L} - \theta L^* + (\lambda_1 + \lambda_2)\theta L^*$. (4.11)

$\overline{c}_1 \geq c_1^*, \overline{c}_2 < c_2^*$ and $\overline{L} \leq \theta L^*$ implies $\overline{R} = 0, \overline{F} = \overline{L}$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L}$. (4.12)

$\overline{c}_1 \geq c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} > \theta L^*$ implies $\overline{R} = \overline{L} - \theta L^*, \overline{F} = \theta L^*$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + \overline{L} - \theta L^* + (\lambda_1 + \lambda_2)\theta L^*$. (4.13)

$\overline{c}_1 \geq c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} \leq \theta L^*$ implies $\overline{R} = 0, \overline{F} = \overline{L}$ and therefore, $\overline{E}_1 + \overline{E}_2 = \overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L}$.

(4.14)

Now we consider every possible combination of the relation between $\overline{c}_1$ and c_1^* and the relation between $\overline{c}_2$ and c_2^* and derive the implications of (4.1) for the relationship between total social costs at $(\overline{c}_1, \overline{c}_2)$ and total social costs at (c_1^*, c_2^*).

Case 1
If $\overline{c}_1 < c_1^*$ and $\overline{c}_2 < c_2^*$ then from (4.2), (4.5) and (4.8) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$
$\rightarrow \overline{c}_1 + \overline{c}_2 + \overline{L} - \theta L^* \leq c_1^* + c_2^*$
$\rightarrow \overline{c}_1 + \overline{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$
and this, in view of the fact that $\theta \leq 1$, implies
$\overline{T} \leq T^*$.

Case 2
If $\overline{c}_1 < c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} \geq L(c_1^*, \overline{c}_2) > \theta L^*$ then from (4.2), (4.6) and (4.9) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^* + L(c_1^*, \overline{c}_2) - \theta L^*$
$\rightarrow \overline{c}_1 + \overline{c}_2 + \overline{L} \leq c_1^* + c_2^* + L(c_1^*, \overline{c}_2)$
which, in view of the fact that $\overline{c}_2 \geq c_2^* \rightarrow L(c_1^*, \overline{c}_2) \leq L^*$ implies
$\overline{T} \leq T^*$.

Case 3
If $\overline{c}_1 < c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} > \theta L^* \geq L(c_1^*, \overline{c}_2)$ then from (4.2), (4.7) and (4.9) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + \lambda_1 L(c_1^*, \overline{c}_2) + \lambda_2 \theta L^*$
$\rightarrow \overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$
$\rightarrow \overline{c}_1 + \overline{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$
which, in view of the fact that $\theta \leq 1$, implies
$\overline{T} \leq T^*$.

Case 4
If $\overline{c}_1 < c_1^*, \overline{c}_2 \geq c_2^*$ and $L(c_1^*, \overline{c}_2) \leq \overline{L} \leq \theta L^*$ then from (4.2), (4.7) and (4.10) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + \lambda_1 L(c_1^*, \overline{c}_2) + \lambda_2 \theta L^*$
which in view of the fact that $L(c_1^*, \overline{c}_2) \leq \overline{L}$ and $0 \leq \lambda_1 \leq 1$ implies
$\overline{c}_1 + \overline{c}_2 + \lambda_2 \overline{L} \leq c_1^* + c_2^* + \lambda_2 \theta L^*$
which in view of the fact that $\overline{L} \leq \theta L^*$ and $0 \leq \lambda_2 \leq 1$ implies
$\overline{c}_1 + \overline{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$.
This in turn implies that $\overline{T} \leq T^*$ since $\theta \leq 1$.

Case 5
If $\overline{c}_1 \geq c_1^*, \overline{c}_2 < c_2^*$ and $\overline{L} \geq L(\overline{c}_1, c_2^*) > \theta L^*$ then from (4.3), (4.5) and (4.11) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$
$\overline{c}_1 + \overline{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$
which, in view of the fact that $\theta \leq 1$, implies
$\overline{T} \leq T^*$.

Case 6

If $\bar{c}_1 \geq c_1^*, \bar{c}_2 < c_2^*$ and $\overline{L} > \theta L^* \geq L(\bar{c}_1, c_2^*)$ then from (4.4), (4.5) and (4.11) it follows that (4.1) implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + \lambda_1 \theta L^* + \lambda_2 L(\bar{c}_1, c_2^*)$

$\rightarrow \bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$

$\rightarrow \bar{c}_1 + \bar{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$

which, in view of the fact that $\theta \leq 1$, implies

$\overline{T} \leq T^*$.

Case 7

If $\bar{c}_1 \geq c_1^*, \bar{c}_1 < c_2^*$ and $L(\bar{c}_1, c_2^*) \leq \overline{L} \leq \theta L^*$ then from (4.4), (4.5) and (4.12) it follows that (4.1) implies

$\rightarrow \bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + \lambda_1 \theta L^* + \lambda_2 L(\bar{c}_1, c_2^*)$

$\rightarrow \bar{c}_1 + \bar{c}_2 + \lambda_1 \overline{L} \leq c_1^* + c_2^* + \lambda_1 \theta L^*$

and this in view of the fact that $\overline{L} \leq \theta L^* \wedge 0 \leq \lambda_1 \leq 1$ implies

$\bar{c}_1 + \bar{c}_2 + \overline{L} \leq c_1^* + c_2^* + \theta L^*$

which, in view of the fact that $0 \leq \theta \leq 1$, implies

$\overline{T} \leq T^*$.

Case 8

If $\bar{c}_1 \geq c_1^*, \bar{c}_2 \geq c_2^*$ and $L(\bar{c}_1, c_2^*), L(c_1^*, \bar{c}_2) \geq \overline{L} > \theta L^*$ then from (4.3), (4.6) and (4.13) it follows that (4.1) implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\theta L^* + \overline{L} - \theta L^* \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^* + L(c_1^*, \bar{c}_2) - \theta L^*$

$\rightarrow \bar{c}_1 + \bar{c}_2 + \overline{L} \leq c_1^* + c_2^* + L(c_1^*, \bar{c}_2)$

which, in view of the fact that $\bar{c}_2 \geq c_2^* \rightarrow L(c_1^*, \bar{c}_2) \leq L^*$, implies

$\overline{T} \leq T^*$.

Case 9

If $\bar{c}_1 \geq c_1^*, \bar{c}_2 \geq c_2^*$ and $L(c_1^*, \bar{c}_2), L(\bar{c}_1, c_2^*) > \theta L^* \geq \overline{L}$ then from (4.3), (4.6) and (4.14) it follows that (4.1) implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + L(c_1^*, \bar{c}_2) - \theta L^* + (\lambda_1 + \lambda_2)\theta L^*$

which in view of the fact that $\overline{L} \leq \theta L^* \wedge 0 \leq \lambda_1 + \lambda_2 \leq 1$ implies

$\bar{c}_1 + \bar{c}_2 + \overline{L} \leq c_1^* + c_2^* + L(c_1^*, \bar{c}_2) - \theta L^* + \theta L^*$

which, in view of the fact that $\bar{c}_2 \geq c_2^* \rightarrow L(c_1^*, \bar{c}_2) \leq L^*$, implies

$\overline{T} \leq T^*$.

Case 10

If $\bar{c}_1 \geq c_1^*, \bar{c}_2 \geq c_2^*$ and $L(\bar{c}_1, c_2^*) > \theta L^* \geq L(c_1^*, \bar{c}_2) \geq \overline{L}$ then from (4.3), (4.7) and (4.14) it follows that (4.1) implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + \lambda_1 L(c_1^*, \bar{c}_2) + \lambda_2 \theta L^*$

which in view of the fact that $L(c_1^*, \bar{c}_2) \leq \theta L^*$ implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$

which, in view of the fact that $\overline{L} \leq \theta L^*$ and $0 \leq \lambda_1 + \lambda_2 \leq 1$, implies

$\overline{T} \leq T^*$.

Case 11

If $\bar{c}_1 \geq c_1^*, \bar{c}_2 \geq c_2^*$ and $L(c_1^*, \bar{c}_2) > \theta L^* \geq L(\bar{c}_1, c_2^*) \geq \overline{L}$ then from (4.4), (4.6) and (4.14) it follows that (4.1) implies

$\bar{c}_1 + \bar{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + L(c_1^*, \bar{c}_2) - \theta L^* + \lambda_1 \theta L^* + \lambda_2 L(\bar{c}_1, c_2^*)$

which, in view of the fact that $L(c_1^*, \overline{c}_2) \leq L^*, \overline{L}, L(\overline{c}_1, c_2^*) \leq \theta L^* \wedge 0 \leq \lambda_1 + \lambda_2 \leq 1$, implies $\overline{T} \leq T^*$.

Case 12
If $\overline{c}_1 \geq c_1^*, \overline{c}_2 \geq c_2^*$ and $\overline{L} \leq L(\overline{c}_1, c_2^*), L(c_1^*, \overline{c}_2) \leq \theta L^*$ then from (4.4), (4.7) and (4.14) it follows that (4.1) implies
$\overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + \lambda_1 L(c_1^*, \overline{c}_2) + \lambda_2 L(\overline{c}_1, c_2^*)$
$\rightarrow \overline{c}_1 + \overline{c}_2 + (\lambda_1 + \lambda_2)\overline{L} \leq c_1^* + c_2^* + (\lambda_1 + \lambda_2)\theta L^*$
which, in view of the fact that $0 \leq \lambda_1 + \lambda_2 \leq 1$, implies
$\overline{T} \leq T^*$.

Thus, in all cases[16] (4.1) implies $\overline{T} \leq T(c_1^*, c_2^*)$. This, in view of the fact that $(c_1^*, c_2^*) \in M$, implies $(\overline{c}_1, \overline{c}_2) \in M$.

□

Proposition 5. *Let* **g** *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule. If* $0 < \theta \leq 1$ *and* $\lambda_1 + \lambda_2 > 1$ *then* **g** *is inefficient.*

Proof. Let **g** be a $((\lambda_1, \lambda_2), \theta)$ - negligence rule such that $0 < \theta \leq 1$ and $\lambda_1 + \lambda_2 > 1$.
Choose δ and v such that $0 < v < \theta\delta$. Note that $0 \leq (1-\theta)\delta$. Choose ϵ_1, ϵ_2 such that $(1-\theta)\delta < \epsilon_1 + \epsilon_2 - v$, $\epsilon_1 < \lambda_1 v + (1-\theta)\delta$ and $\epsilon_2 < \lambda_2 v$.[17] Now choose positive numbers ξ_1, ξ_2 and ϵ such that $\epsilon_1 < \xi_1 < \epsilon$ and $\epsilon_2 < \xi_2 < \epsilon$.

Let $C_1 = \{0, \epsilon_1, 2\epsilon_1\}, C_2 = \{0, \epsilon_2, 2\epsilon_2\}$ and let the expected loss function, $L(c_1, c_2)$, be as given in Table 7.

		c_2: 0	ϵ_2	$2\epsilon_2$
c_1	0	$\delta + 2\epsilon$	$\delta + \epsilon$	$\delta + \xi_1$
	ϵ_1	$\delta + \epsilon$	δ	δ
	$2\epsilon_1$	$\delta + \xi_2$	δ	$\theta\delta - v$

TABLE 7. ω_2: Expected Loss

		c_2: 0	ϵ_2	$2\epsilon_2$
c_1	0	$\delta + 2\epsilon$	$\delta + \epsilon + \epsilon_2$	$\delta + \xi_1 + 2\epsilon_2$
	ϵ_1	$\delta + \epsilon + \epsilon_1$	$\delta + \epsilon_1 + \epsilon_2$	$\delta + \epsilon_1 + 2\epsilon_2$
	$2\epsilon_1$	$\delta + \xi_2 + 2\epsilon_1$	$\delta + 2\epsilon_1 + \epsilon_2$	$\theta\delta + 2\epsilon_1 + 2\epsilon_2 - v$

TABLE 8. ω_2: Total Social Costs

Table 8 gives the total social costs for every possible configuration of costs of care. Here $M = \{(\epsilon_1, \epsilon_2)\}$. Let $(c_1^*, c_2^*) = (\epsilon_1, \epsilon_2)$. Note that $\{0, \epsilon_1, 2\epsilon_1\}, \{0, \epsilon_2, 2\epsilon_2\}, L$ and (ϵ_1, ϵ_2) constitute an application in Ω. We shall refer to this application as ω_2.

[16]Cases 1-12 constitute the exhaustive and mutually exclusive list of possibilities with respect to combinations of the relation between $\overline{c}_1$ and c_1^* and; $\overline{c}_2$ and c_2^*. Note that (4.2), (4.6) and (4.10) cannot hold together. Similarly, (4.3), (4.5) and (4.12); (4.3), (4.7) and (4.13); (4.4), (4.6) and (4.13); (4.4), (4.7) and (4.13) cannot hold together.
[17]Note that if $\lambda_1 + \lambda_2 \leq 1$ then $(1-\theta)\delta < \epsilon_1 + \epsilon_2 - v$ would contradict $\epsilon_1 < \lambda_1 v + (1-\theta)\delta$ and $\epsilon_2 < \lambda_2 v$.

$R(2\epsilon_1, 2\epsilon_2) = 0; R(2\epsilon_1, \epsilon_2) = R(\epsilon_1, 2\epsilon_2) = (1-\theta)\delta; R(2\epsilon_1, 0) = (1-\theta)\delta + \xi_2, R(0, 2\epsilon_2) = (1-\theta)\delta + \xi_1; F(2\epsilon_1, 2\epsilon_2) = \theta\delta - v; F(2\epsilon_1, \epsilon_2) = F(\epsilon_1, 2\epsilon_2) = F(2\epsilon_1, 0) = F(0, 2\epsilon_2) = \theta\delta$.

$E_1(2\epsilon_1, 2\epsilon_2) = 2\epsilon_1 + x_1^1 R(2\epsilon_1, 2\epsilon_2) + \lambda_1 F(2\epsilon_1, 2\epsilon_2) = 2\epsilon_1 + \lambda_1(\theta\delta - v)$;
$E_1(\epsilon_1, 2\epsilon_2) = \epsilon_1 + x_1^1 R(\epsilon_1, 2\epsilon_2) + \lambda_1 F(\epsilon_1, 2\epsilon_2) = \epsilon_1 + (1-\theta)\delta + \lambda_1\theta\delta$;
$E_1(0, 2\epsilon_2) = x_1(0,1) R(0, 2\epsilon_2) + \lambda_1 F(0, 2\epsilon_2) = (1-\theta)\delta + \xi_1 + \lambda_1\theta\delta$.
Note that $E_1(0, 2\epsilon_2) > E_1(\epsilon_1, 2\epsilon_2)$.

$E_2(2\epsilon_1, 2\epsilon_2) = 2\epsilon_2 + x_2^1 R(2\epsilon_1, 2\epsilon_2) + \lambda_2 F(2\epsilon_1, 2\epsilon_2) = 2\epsilon_2 + \lambda_2(\theta\delta - v)$;
$E_2(2\epsilon_1, \epsilon_2) = \epsilon_2 + x_2^1 R(2\epsilon_1, \epsilon_2) + \lambda_2 F(2\epsilon_1, \epsilon_2) = \epsilon_2 + \lambda_2\theta\delta$;
$E_2(2\epsilon_1, 0) = x_2(1,0) R(2\epsilon_1, 1) + \lambda_2 F(2\epsilon_1, 0) = (1-\theta)\delta + \xi_2 + \lambda_2\theta\delta$.
Note that $E_2(2\epsilon_1, 0) > E_2(2\epsilon_1, \epsilon_2)$.

$2\epsilon_1 + \lambda_1(\theta\delta - v) - \epsilon_1 - (1-\theta)\delta - \lambda_1\theta\delta = \epsilon_1 - (1-\theta)\delta - \lambda_1 v < 0$. Thus $E_1(2\epsilon_1, 2\epsilon_2) < E_1(\epsilon_1, 2\epsilon_2), E_1(0, 2\epsilon_2)$.
$2\epsilon_2 + \lambda_2(\theta\delta - v) - \epsilon_2 - \lambda_2\theta\delta = \epsilon_2 - \lambda_2 v < 0$. Thus $E_2(2\epsilon_1, 2\epsilon_2) < E_2(2\epsilon_1, \epsilon_2), E_2(2\epsilon_1, 0)$.
Therefore $(2\epsilon_1, 2\epsilon_2)$ is a Nash equilibrium of $(\mathbf{g}, \omega_2)$.
This establishes that $\mathbf{g}$ is not efficient. □

Theorem 2. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule. Suppose* $0 < \theta \leq 1$. $\mathbf{g}$ *is efficient iff* $\lambda_1 + \lambda_2 \leq 1$.

Proof. Immediate from Propositions 3, 4 and 5. □

Theorem 3. *Let* $\mathbf{g}$ *be a* $((\lambda_1, \lambda_2), \theta)$*-negligence rule. If* $\theta > 1$ *then* $\mathbf{g}$ *is inefficient.*

Proof. Let $\mathbf{g}$ be a $((\lambda_1, \lambda_2), \theta)$-negligence rule and let $\theta > 1$. Choose $\delta > 0$. Let $\xi = \frac{(\theta-1)\delta}{4}$ and $\epsilon = \frac{3(\theta-1)\delta}{10}$. Let $C_1 = C_2 = \{0, \epsilon\}$ and let the expected loss function, $L(c_1, c_2)$, be as given in Table 9.

c_1 \ c_2	0	ϵ
0	$\theta\delta - \xi$	$\theta\delta - 2\xi$
ϵ	$\theta\delta - 2\xi$	δ

TABLE 9. ω_3: Expected Loss

c_1 \ c_2	0	ϵ
0	$\theta\delta - \xi$	$\theta\delta - 2\xi + \epsilon$
ϵ	$\theta\delta - 2\xi + \epsilon$	$\delta + 2\epsilon$

TABLE 10. ω_3: Total Social Costs

Table 10 gives the total social costs for every possible configuration of costs of care. Here $M = \{(\epsilon, \epsilon)\}$. Let $(c_1^*, c_2^*) = (\epsilon, \epsilon)$. Note that $C_1 = C_2 = \{0, \epsilon\}, L$ and (ϵ, ϵ) constitute an application in Ω. We shall refer to this application as ω_3.
Now, $\theta\delta - \xi - \theta L^* < 0$. Therefore, $R(c_1, c_2) = 0$ and $F(c_1, c_2) = L(c_1, c_2)$ for all $(c_1, c_2) \in C_1 \times C_2$. $E_1(0,0) = \lambda_1(\theta\delta - \xi), E_2(0,0) = \lambda_2(\theta\delta - \xi)$ and $E_1(\epsilon, 0) = \epsilon + \lambda_1(\theta\delta - 2\xi), E_2(0, \epsilon) = \epsilon + \lambda_2(\theta\delta - 2\xi)$. $\lambda_i(\theta\delta - \xi) - \epsilon - \lambda_i(\theta\delta - 2\xi) = \lambda_i\xi - \epsilon \leq \xi - \epsilon < 0$. Thus, $E_1(0,0) < E_1(\epsilon, 0)$ and $E_2(0,0) < E_2(0, \epsilon)$ and therefore, (0, 0) is a Nash equilibrium of $(\mathbf{g}, \omega_3)$.
This establishes that g is not efficient. □

The logic behind the results is quite simple. First note that even when both parties take due care the expected loss is equal to optimal expected loss (L^*). Therefore, internalization of the expected loss over and above the optimal expected loss ($L - L^*$) and not the entire expected loss is crucial for efficiency. Given that a $((\lambda_1, \lambda_2), \theta)$-negligence rule assigns the entire excess loss to the injurer if she is negligent and to the victim otherwise, $L - L^*$ is internalized by the interacting parties iff the expected excess loss is at least as large as $L - L^*$ and the assignment of the specified loss does not distort incentives. It may be noted that the excess loss is at least as large as $L - L^*$ iff $0 \leq \theta \leq 1$.

If $\theta > 1$ then the expected excess loss is less than $L - L^*$ and the assignment of a part of $L - L^*$ is done in fixed proportions. As a result, parties do not internalize the full benefit of a reduction in expected loss due to greater care on their part. This may induce both parties to choose care levels which are less than what is due from them even when such care configurations do not minimize total social costs.

If $0 < \theta \leq 1$ and $\lambda_1 + \lambda_2 > 1$ then the liabilities for the specified loss add up to more than the specified loss. Therefore a reduction in expected loss may bring about a reduction in the expected magnitude of the total liabilities for specified loss which is greater than the reduction in the expected specified loss itself. This may induce both parties to choose care levels which are greater than what is due from them even when such care configurations are not total social costs minimizing.

If $\theta = 0$ then the specified loss is 0 and entire loss is assigned to the injurer if she is negligent and to the victim otherwise. This results in internalization of the entire loss by the victim and the injurer. If $0 < \theta \leq 1$ and $\lambda_1 + \lambda_2 \leq 1$ then the expected excess loss, which is assigned to the injurer if she is negligent and to the victim otherwise, is at least as large as $L - L^*$; and the two parties together bear the specified loss (or a part thereof). Thus, the assignment of the specified loss does not distort incentives and the two parties internalize at least $L - L^*$.

Thus it turns out that a $((\lambda_1, \lambda_2), \theta)$-negligence rule is efficient if and only if the specified loss is such that its expected value is not greater than the loss at the levels of care due from the two parties and the shares of the specified loss to be borne by the two parties add upto less than or equal to specified loss.

4. Discussion

An important implication of our result is that an efficient $((\lambda_1, \lambda_2), \theta)$-negligence rule may assign only a part of the specified loss to the interacting parties for all possible care configuration and can therefore result in an assignment of liabilities which is always decoupled. This is in sharp contrast to earlier results which demonstrate the inconsistency of complete decoupling and efficiency in the assignment of liabilities.[18] In particular, note that the rule which assigns none of the expected optimal loss to any of the two parties and assigns the entire loss over and above the expected optimal loss to the injurer if she is negligent and to the victim otherwise is an efficient rule. Such a rule can potentially be very important in the context of interactions in which neither of the parties may be in positions to bear a large burden of the loss. An injurer who doesn't have the means to pay for the losses that he causes can turn out to be insolvent and judgement-proof. [19] Under the standard negligence rule, potentially judgement-proof injurers will not fully internalize the losses. If the expected optimal loss is large and the injurer is in a position to bear the loss in excess of the optimal excess loss but not the full loss then appropriate incentives can be created under a (λ, θ)-negligence rule by making the injurer liable only for the loss is excess of the optimal excess loss if and only if he is negligent. Thus, under a (λ, θ)-negligence rule, efficiency can be achieved in such a situation only at the cost of the victim and this cost can be significant for a victim who, like the injurer, is also wealth constrained. If the expected optimal loss is large and both parties involved in the interaction are in a position to bear the loss over and above the expected optimal loss individually but not the full loss even together then appropriate incentives can be created under a $((\lambda_1, \lambda_2), \theta)$-negligence

[18]See Jain[11], Kaur and Kundu[16] and Kundu and Kaur[18]

[19]See Summers[1] and Shavell[24] for an analysis of the judgement-proof problem.

rule by compensating the victim for a part of the loss (upto the optimal expected loss) from a special fund set up for the purpose. Thus, in these conditions, a((λ_1, λ_2), θ)-negligence rule can result in an efficient outcome when the standard negligence rule or any of the (λ, θ)-negligence rules fail to do so.

The import of our analysis goes much beyond the possibility of an efficiency improvement (over other rules discussed in the literature) in situations involving potentially judgement-proof parties and lies in a significant broadening of the scope for non-efficiency consideration. It follows from our result that a $((\lambda_1, \lambda_2)$, $\theta)$-negligence rule can achieve efficiency even after subsidising a part of the specified loss (where the expected value of the specified loss is capped at the expected optimal loss) and apportioning the remaining part of the specified loss between the two parties in any fixed proportions on the basis of non efficiency considerations. Not only does such a rule have room for considerations of justice and fairness it also indicates a role of the welfare state in creating efficient incentives in the context of interactions in which both parties are wealth constrained. Thus, an appropriate combination of decomposition of the actual loss and a decoupling of liabilities for the specified loss may lead to better assignment of liabilities both from the point of view of efficiency and non-efficiency concerns.

5. Conclusion

The paper generalizes the notion of a (λ, θ)-negligence rule to incorporate the possibility of assignment of liabilities which is not coupled always and to examine the requirements of efficiency on different parts of the accidental loss and also the relationship between decoupling and efficiency. Our results indicate that the assignment of liabilities for both parts of the loss are relevant from the point of view of efficiency for different reasons. Our results also indicate that efficiency can be achieved even with an assignment of liabilities which is completely decoupled thereby opening up the possibility of improving efficiency when the interacting parties may not be in a position to bear the entire loss.

The rules analysed here apportion the excess loss in a particular manner: the excess loss is assigned to the injurer if and only if she is negligent and to the victim otherwise. As a result the assignment of liabilities for the excess loss alone is coupled and the decoupling of the overall loss can arise only due to the decoupled assignment of the liabilities for the specified loss. In principle, it is possible to define a more general notion of a rule by allowing all possible assignments of the excess loss based on the level of nonnegligence of the parties involved. Under such rules a decoupling could result from decoupled assignment of the liabilities for the excess loss alone or in conjunction with the assignment of the liabilities for the specified loss. It would be interesting to see if the the results of the paper hold for this more general class of rules.

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